Prerequisite Skills

1. Terms that have the same variable part are called?

2. Simplify the Expression.
   (1) $3x + (-6x)$

   (2) $5 + 4x + 2$

   (3) $4(2x - 1) + x$

   (4) $-(x + 4) - 6x$

3. Simplify the expression.
   (1) $(3xy)^3$

   (2) $xy^2 \cdot xy^3$

   (3) $(x^5)^3$

   (4) $(-x)^3$

4. Find the greatest common factor of the pair of numbers.
   (1) 121, 77

   (2) 96, 32

   (3) 81, 42

   (4) 12, 56

8.1 ADD AND SUBTRACT POLYNOMIALS

A **monomial** is a number, a variable, or the product of a number and one or more variables with whole number exponents. The **degree of a monomial** is the sum of the exponents of the variables in the monomial. The degree of a nonzero constant term is 0. The constant 0 does not have a degree.

Monomial	**Degree**
10	0
$3x$	1
$\frac{1}{2}ab^2$	$1+2=3$
$-1.8m^5$	5

Not a monomial	**Reason**
$5+x$	A sum is not a monomial.
$\frac{2}{n}$	A monomial cannot have a variable in the denominator.
4^a	A monomial cannot have a variable exponent.
x^{-1}	The variable must have whole number exponent.

A **polynomial** is a monomial or a sum of monomials, each called a *term* of the polynomial. The **degree of a polynomial** is the greatest degree of its terms.

When a polynomial is written so that the exponents of a variable decrease from left to right, the coefficient of the first term is called the **leading coefficient**.

Leading coefficient　　Degree　　Constant term

$$2x^3 + x^2 - 5x + 12$$

Ex1) Rewrite the expression so that the exponents decrease from left to right. Identify the degree and leading coefficient of the polynomial.

(1) $15x - x^3 + 3$

(2) $5y - 2y^2 + 9$

BINOMIALS AND TRINOMIALS

A polynomial with two terms is called a **binomial**. A polynomial with three terms is called a **trinomial**.

Ex2) Tell whether the expression is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise, tell why it is not a polynomial.

Expression	Is it a polynomial?	Classify by degree and number of terms
9		
$2x^2 + x - 5$		
$6n^4 - 8^n$		
$n^{-2} - 3$		
$7bc^3 + 4b^4c$		
$y^3 - 4y + 3$		

ADDING POLYNOMIALS

To add polynomials, add like terms. You can use a vertical or a horizontal format.

Ex3) Find the sum.

(1) $(2x^3 - 5x^2 + x) + (2x^2 + x^3 - 1)$

(2) $(3x^2 + x - 6) + (x^2 + 4x + 10)$

(3) $(5x^3 + 4x - 2x) + (4x^2 + 3x^3 - 6)$

SUBTRACTING POLYNOMIALS

To subtract a polynomial, add its opposite. To find the opposite of a polynomial, multiply each of its terms by −1.

Ex4) Find the difference.

(1) $(4n^2 + 5) - (-2n^2 + 2n - 4)$

(2) $(4x^2 - 3x + 5) - (3x^2 - x - 8)$

(3) $(4x^2 - 7x) - (5x^2 + 4x - 9)$

Ex5) Word Problem

Major League Baseball teams are divided into two leagues. During the period 1995−2001, the attendance *N* and *A* (in thousands) at National and American League baseball games, respectively, can be modeled by

$N = -488t^2 + 5430t + 24{,}700$ and

$A = -318t^2 + 3040t + 25{,}600$

where *t* is the number of years since 1995. About how many people attended Major League Baseball games in 2001?

8.2 MULTIPLY POLYNOMIALS

The diagram shows that a rectangle with width x and length $2x^2 + 3x$. You can also find this product by using the distributive property.

$$x(2x + 3) = x(2x) + x(3) = 2x^2 + 3x$$

In this lesson, you will learn several methods for multiplying polynomials. Each method is based on the distributive property.

$2x + 3$

	x	x	1	1	1
x	x^2	x^2	x	x	x

Ex1) Find the product.

(1) $2x^3(x^3 + 3x^2 - 2x + 5)$

(2) $x(7x^2 + 4)$

Ex2) Find the product by using a table.

(1) $(x - 4)(3x + 2)$

	$3x$	2
x		
-4		

(2) $(a + 3)(2a + 1)$

(3) $(4n - 1)(n + 5)$

Ex3) Find the product vertically and horizontally.

(1) $(2x^2 + 5x - 1)(4x - 3)$

(2) $(b^2 + 6b - 7)(3b - 4)$

(3) $(x^2 + 2x + 1)(x + 2)$

(4) $(3y^2 - y + 5)(2y - 3)$

(5) $(4b - 5)(b - 2)$

FOIL PATTERN

The letters of the word FOIL can help you to remember how to use the distributive property to multiply binomials. The letters should remind you of the words **F**irst, **O**uter, **I**nner, and **L**ast.

$$(2x + 3)(4x + 1) = 8x^2 + 2x + 12x + 3$$

Ex4) Find the product using the FOIL pattern.

(1) $(3a + 4)(a - 2)$

(2) $(x^2 + 2x + 1)(x + 2)$

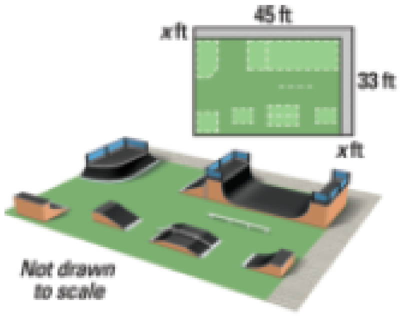
(3) $(3y^2 - y + 5)(2y - 3)$

(4) $(4b - 5)(b - 2)$

Ex5) The dimensions of a rectangle are $x+3$ and $x+2$. Which expression represents the area of the rectangle?

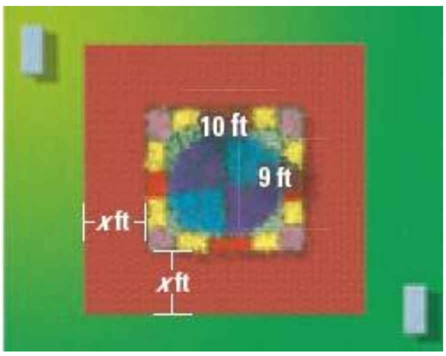
(1) x^2+6
(2) x^2+5x+6
(3) x^2+6x+6
(4) x^2+6x

Ex6) You are designing a rectangular skateboard park on a lot that is on the corner of a city block. The park will have a walkway along two sides. The dimensions of the lot and the walkway are shown in the diagram.



- Write a polynomial that represents the area of the skateboard park
- What is the area of the park if the walkway is 3 feet wide?

Ex7) You are planning to build a walkway that surrounds a rectangular garden, as shown. The width of the walkway around the garden is the same on every side.



a. Write a polynomial that represents the combined area of the garden and the walkway.

b. Find the combined area when the width of the walkway is 4 feet.

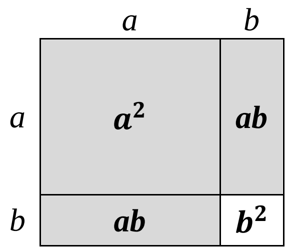
8.3 FIND SPECIAL PRODUCTS OF POLYNOMIALS

The diagram shows a square with a side length of $(a + b)$ units. You can see that the area of the square is

$$(a + b)^2 = a^2 + 2ab + b^2.$$

This is one version of a pattern called the square of a binomial. To find another version of this pattern, use algebra: replace b with $-b$.

$$\big(a + (-b)\big)^2 = a^2 + 2a(-b) + (-b)^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$



Replace b with $-b$ in the pattern above. Simplify.

Square of a Binomial Pattern

Algebra	Example
$(a + b)^2 = a^2 + 2ab + b^2$	$(x + 5)^2 = x^2 + 10x + 25$
$(a - b)^2 = a^2 - 2ab + b^2$	$(2x - 3)^2 = 4x^2 - 12x + 9$

Ex1) Find the product.

a. $(3x + 4)^2$

b. $(5x - 2y)^2$

c. $(x + 3)^2$

d. $(4y - y)^2$

e. $(3m + n)^2$

f. $(4y + 2z)^2$

SUM AND DIFFERENCE PATTERN

To find the product $(x+2)(x-2)$, you can multiply the two binomials using the FOIL pattern.

$(x+2)(x-2) = x^2 - 2x + 2x - 4$ Use FOIL pattern.

$= x^2 - 4$ Combine like terms.

Sum and Difference Pattern

Algebra	Example
$(a+b)(a-b) = a^2 - b^2$	$(x+3)(x-3) = x^2 - 9$

Ex2) Find the product

a. $(t+5)(t-5)$

b. $(3x+y)(3x-y)$

c. $(x+10)(x-10)$

d. $(2x+1)(2x-1)$

e. $(x+3y)(x-3y)$

SPECIAL PRODUCTS AND MENTAL MATH

The special product patterns can help you use mental math to find certain products of numbers.

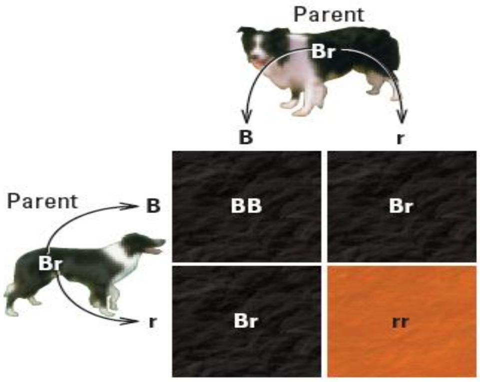
Ex3) Use special products and mental math.

a. $26 \cdot 34$

b. $19 \cdot 21$

c. $8 \cdot 12$

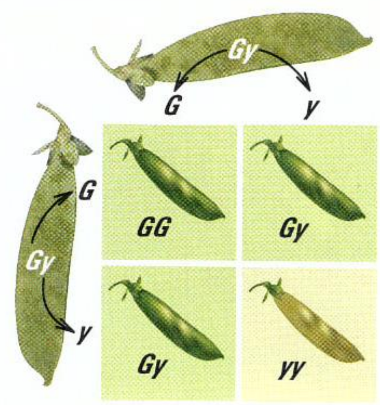
Ex4) The gene B is for black patches and the gene r is for red patches. Any gene combination with a B results in black patches. Suppose each parent has the same gene combination Br. The Punnett square shows the possible gene combinations of the offspring and the resulting patch color.



- What percent of the possible gene combinations of the offspring result in black patches?

- Show how you could use a polynomial to model the possible gene combinations of the offspring.

Ex6) In pea plants, the gene G is for green pods, and the gene y for yellow pods. Any gene combination with a G results in a green pod. Suppose two pea plants have the same gene combination Gy. The Punnett square shows the possible gene combinations of an off spring pea plant and the resulting pod color.



- What percent of possible gene combinations of the offspring plant result in a yellow pod?

- Show how you could use a polynomial to model the possible gene combinations of the offspring.

8.4 SOLVE POLYNOMIAL EQUATIONS IN FACTORED FORM

For any real number a, $a \cdot 0 = 0$. This is equivalent to saying:

For real numbers a and b, if $a = 0$ or $b = 0$, then $ab = 0$.

The converse of this statement is also true, and it is called the zero-product property.

Zero-Product Property

Let a and b be real numbers. If $ab = 0$, then $a = 0$ or $b = 0$.

The zero-product property is used to solve an equation when one side is zero and the other side is a product of polynomial factors. The solutions of such an equation are also called **roots.**

Ex1) Use the zero-product property.

1. $(x - 4)(x + 2) = 0$

2. $(x - 5)(x - 1) = 0$

FACTORING

To solve a polynomial equation using the zero-product property, you may need to *factor* the polynomial, or write it as a product of other polynomials. Look for the *greatest common factor* (GCF) of the polynomial's terms. This is a monomial with an integer coefficient that divides evenly into each term.

Ex2) Factor out the greatest common monomial factor.

1. $12x + 42y$

2. $4x^4 + 24x^3$

3. $14m + 35n$

Ex3) Solve an equation by factoring.

1. $2x^2 + 8x = 0$

2. $6n^2 = 15n$

3. $a^2 + 5a = 0$

4. $3s^2 - 9s = 0$

5. $4x^2 = 2x$

VERTICAL MOTION

A *projectile* is an object that is propelled into the air but has no power to keep itself in the air. A thrown ball is a projectile, but an airplane is not. The height of a projectile can be described by the **vertical motion model.**

The height *h* (in feet) of a projectile can be modeled by

$$\boldsymbol{h = -16t^2 + vt + s}$$

where *t* is the time (in seconds) the object has been in the air, *v* is the initial vertical velocity (in feet per second), and *s* is the initial height (in feet).

Ex4) A startled armadillo jumps straight into the air with an initial vertical velocity of 14 feet per second. After how many seconds does it land on the ground?

8.5 Factor $x^2 + bx + c$

From lesson 8.2, you know that

$$(x+3)(x+4) = x^2 + (3+4)x + 3 \cdot 4 = x^2 + 7x + 12$$

You will reverse this process to factor trinomials of the form $x^2 + bx + c$.

Factoring $x^2 + bx + c$	
Algebra	$x^2 + bx + c = (x+p)(x+q)$ provided $p + q = b$ and $pq = c$.
Example	$x^2 + 5x + 6 = (x+3)(x+2)$ provided $3 + 2 = 5$ and $3 \cdot 2 = 6$.

Ex1) Factor the trinomial.

1. $x^2 + 11x + 18$

2. $x^2 + 3x + 2$

3. $a^2 + 7a + 10$

4. $t^2 + 9t + 14$

FACTORING

When factoring a trinomial, first consider the signs of p and q.

$(x+p)(x+q)$	$x^2 + bx + c$	**Signs of b and c**
$(x+2)(x+3)$	$x^2 + 5x + 6$	b is positive; c is positive.
$(x+2)\big(x+(-3)\big)$	$x^2 - x - 6$	b is negative; c is negative.
$(x+(-2))(x+3)$	$x^2 + x - 6$	b is positive; c is negative.
$(x+(-2))\big(x+(-3)\big)$	$x^2 - 5x + 6$	b is negative; c is positive.

By observing the signs of b and c in the table, you can see that:

- b and c are positive when both p and q are positive.
- b is negative and c is positive when both p and q are negative.
- c is negative when p and q have different signs.

Ex2) Factor the trinomials.

1. $n^2 - 6n + 8$

2. $y^2 + 2y - 15$

3. $x^2 - 4x + 3$

4. $t^2 - 8t + 12$

5. $m^2 + m - 20$

6. $w^2 + 6w - 16$

7. $z^2 + 8z - 48$

Ex3) Solve a polynomial equation

1. $x^2 + 3x = 18$

2. $s^2 - 2s = 24$

3. $x^2 - 10x + 21 = 0$

4. $x^2 - 7x - 120 = 0$

5. $c^2 + 15c = -44$

6. $-x^2 + 16x = 28$

7. $x^2 - 14x - 39 = 12$

8. $x^2 + 10x - 50 = -11$

9. $x^2 + 5x - 2 = -8$

Ex4) Factor a trinomial in two variables

1. $x^2 + 8xy + 16y^2$

2. $d^2 - 6dz + 5z^2$

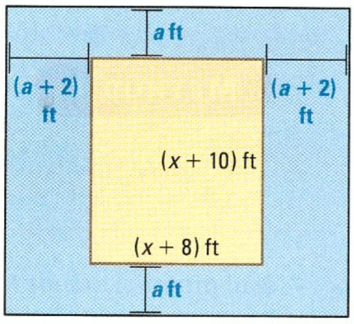
3. $a^2 + 2ab - 15b^2$

4. $g^2 + 4gh - 60h^2$

Ex5) You are making banners to hang during school spirit week. Each banner requires 16.5 square feet of felt and will be cut as shown. Find the width of one banner.

Ex6) A rectangular stage is positioned in the center of a rectangular room, as shown. The area of the stage is 120 square feet.

a. Use the dimensions given in the diagram to find the length and width of the stage.



b. the combined area of the stage and the surrounding floor is 360 square feet. Find the length and width of the room.

8.6 FACTOR $ax^2 + bx + c$

When factoring a trinomial of the form $ax^2 + bx + c$, first consider the signs of b and c, as in Lesson 8.5. This approach works when a is positive.

Ex1) Factor the trinomials.

a. $2x^2 - 7x + 3$

- Because b is negative and c is positive, both factors of c must be negative. Make a table to organize your work.
- You must consider the order of the factors of 3, because the x-terms of the possible factorizations are different.

Factors of 2	Factors of 3	Possible factorization	Middle term when multiplied	
1, 2	$-1, -3$	$(x-1)(2x-3)$	$-3x - 2x = -5x$	X
1, 2	$-3, -1$	$(x-3)(2x-1)$	$-x - 6x = -7x$	O

- $2x^2 - 7x + 3 = (x-3)(2x-1)$

b. $3n^2 + 14n - 5$

- Because b is positive and c is negative, the factors of c have different signs.

Factors of 3	Factors of -5	Possible factorization	Middle term when multiplied	
1, 3	1, -5	$(n+1)(3n-5)$	$-5n + 3n = -2n$	X
1, 3	-1, 5	$(n-1)(3n+5)$	$5n - 3n = 2n$	X
1, 3	5, -1	$(n+5)(3n-1)$	$-n + 15n = 14n$	O
1, 3	-5, 1	$(n-5)(3n+1)$	$n - 15n = -14n$	X

- $3n^2 + 14n - 5 = (n+5)(3n-1)$

c. $3t^2 + 8t + 4$

d. $4s^2 - 9s + 5$

e. $2h^2 + 13h - 7$

f. $-4n^2 + 12n + 7$

- Factor -1 from each term of the trinomial.
  $-4n^2 + 12n + 7 = -(4n^2 - 12n - 7)$
- Factor the trinomial $4n^2 - 12n - 7$. Because b and c are both negative, the factors of c must have different signs. As in the previous examples, use a table to organize information about the factors of a and c.

Factors of 4	Factors of -7	Possible factorization	Middle term when multiplied	
1, 4	1, -7	$(n+1)(4n-7)$	$-7n + 4n = -3n$	X
1, 4	7, -1	$(n+7)(4n-1)$	$-n + 28n = 27n$	X
1, 4	-1, 7	$(n-1)(4n+7)$	$7n - 4n = 3n$	X
1, 4	-7, 1	$(n-7)(4n+1)$	$n - 28n = -27n$	X
2, 2	1, -7	$(2n+1)(2n-7)$	$-14n + 2n = -12n$	O
2, 2	-1, 7	$(2n-1)(2n+7)$	$14n - 2n = 12n$	X

- $-4x^2 + 12x + 7 = -(2x+1)(2x-7)$

g. $-2y^2 - 5y - 3$

h. $-5m^2 + 6m - 1$

i. $-3x^2 - x + 2$

FINDING A COMMON FACTOR

In Lesson 8.4, you learned to factor out the greatest common monomial factor from the terms of a polynomial. Sometimes you may need to do this before finding two binomial factors of a trinomial.

Ex2) An athlete throws a discus from an initial height of 6 feet and with an initial vertical velocity of 46 feet per second. (Vertical motion model: $h = -16t^2 + vt + s$)

a. Write an equation that gives the height (in feet) of the discus as a function of the time (in seconds) function of the time (in seconds) since it left the athlete's hand.

b. After how many seconds does the discus hit the ground?

Ex3) in a shot put event, an athlete throws the shot put from an initial height of 6 feet and with an initial vertical velocity of 29 feet per second. After how many seconds hoes the shot put hit the ground?

Ex4) A rectangle's length is 13 meters more than 3 times its width. The area is 10 square meters. What is the width?

Ex5) A rectangle's length is 1 inch more than twice its width. The area is 6 square inches. What is the width?

Ex6) The Parthenon in Athens, Greece, is an ancient structure that has a rectangular base. The length of the Parthenon's base is 8 meters more than twice its width. The area of the base is about 2170 square meters. Find the length and width of the Parthenon's base.

Ex7) A diver dives from a cliff when her center of gravity is 46 feet above the surface of the water. Her initial vertical velocity leaving the cliff is 9 feet per second. After how many seconds does her center of gravity enter the water?

8.7 FACTOR SPECIAL PRODUCTS

You can use the special product patterns you studied in Lesson 8.3 to factor polynomials, such as the difference of two squares.

Difference of Two Squares Pattern

Algebra	**Example**
$a^2 - b^2 = (a+b)(a-b)$	$4x^2 - 9 = (2x)^2 - 3^2 = (2x+3)(2x-3)$

Ex1) Factor the polynomial.

a. $y^2 - 16$

b. $25m^2 - 36$

c. $x^2 - 49y^2$

d. $8 - 18n^2$

e. $4y^2 - 64$

PERFECT SQUARE TRINOMIALS

The pattern for finding the square of a binomial gives you the pattern for factoring trinomials of the form $a^2 + 2ab + b^2$ and $a^2 - 2ab + b^2$. These are called **perfect square trinomials**.

Perfect Square Trinomial Pattern	
Algebra	**Example**
$a^2 + 2ab + b^2 = (a+b)^2$	$x^2 + 6x + 9 = x^2 + 2(x \cdot 3) + 3^2 = (x+3)^2$
$a^2 - 2ab + b^2 = (a-b)^2$	$x^2 - 10x + 25 = x^2 - 2(x \cdot 5) + 5^2 = (x-5)^2$

Ex2) Factor the polynomial.

a. $n^2 - 12n + 36$

b. $9x^2 - 12x + 4$

c. $4s^2 + 4st + t^2$

d. $-3y^2 + 36y - 108$

e. $h^2 + 4h + 4$

f. $2y^2 - 20y + 50$

g. $3x^2 + 6xy + 3y^2$

Ex3) Solve a polynomial equation

a. $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$

b. $a^2 + 6a + 9 = 0$

c. $w^2 - 14w + 49 = 0$

d. $n^2 - 81 = 0$

Ex4) A window washer drops a wet sponge from a height of 64 feet. After how many seconds does the sponge land on the ground? ($h = -16t^2 + vt + s$)

Ex5) A window washer drops a wet sponge from a height of 16 feet. After how many seconds does the sponge land on the ground?

8.8 FACTOR POLYNOMIALS COMPLETELY

You have used the distributive property to factor a greatest common monomial from a polynomial. Sometimes, you can factor out a common binomial.

Ex1) Factor the expression.

a. $2x(x+4)-3(x+4)$

b. $3y^2(y-2)+5(2-y)$

c. $x(x-2)+(x-2)$

Ex2) Factor the polynomial by grouping.

a. $x^3+3x^2+5x+15$

b. $y^2+y+yx+x$

c. a^3+3a^2+a+3

d. $y^2 + 2x + yx + 2y$

e. $8x^2 + 10x - 3$

FACTORING COMPLETELY

You have seen that the polynomial $x^2 - 1$ can be factored as $(x + 1)(x - 1)$. This polynomial is factorable. Notice that the polynomial $x^2 + 1$ cannot be written as the product of polynomials with integer coefficients. This polynomial is unfactorable. A factorable polynomial with integer coefficients is **factored completely** if it is written as a product of unfactorable polynomials with integer coefficients.

CONCEPT SUMMARY	
Guidelines for Factoring Polynomials completely	
To factor a polynomial completely, you should try each of these steps.	
1. Factor out the greatest common monomial factor.	$3x^2 + 6x = 3x(x + 2)$
2. Look for a difference of two squares or a perfect square trinomial.	$x^2 + 4x + 4 = (x + 2)^2$
3. Factor a trinomial of the form $ax^2 + bx + c$ into a product of binomial factors.	$3x^2 - 5x - 2 = (3x + 1)(x - 2)$
4. Factor a polynomial with four terms by grouping.	$x^3 + x - 4x^2 - 4 = (x^2 + 1)(x - 4)$

Ex3) Factor the polynomial completely.

a. $n^2 + 2n + 1$

b. $4x^2 - 44x + 96$

c. $50h^4 - 2h^2$

d. $3x^3 - 12x$

e. $2y^3 - 12y^2 + 18y$

f. $m^3 - 2m^2 - 8m$

Ex4) Solve a polynomial equation.

a. $3x^3 + 18x^2 = -24x$

b. $w^3 - 8w^2 + 16w = 0$

c. $x^3 - 25x = 0$

d. $c^3 - 7c^2 + 12c = 0$

Ex5) A terrarium in the shape of a rectangular prism has a volume of 4608 cubic inches. Its length is more than 10 inches. The dimensions of the terrarium are shown. Find the length, width, and height of the terrarium.

Ex6) A box in the shape of a rectangular prism has a volume of 72 cubic feet. The box has a length of x feet, a width of $(x - 1)$ feet, and a height of $(x + 9)$ feet. Find the dimensions of the box.

Ex7) A plastic cube is used to display an autographed baseball. The cube has an outer surface area of 54 square inches.

a. What is the length of an outer edge of the cube?

b. What is the greatest volume the cube can possibly have?